Quantum Decoherence Modulated by Special Relativity

Jian-Ming Cai[1], Zheng-Wei Zhou[1],* Ye-Fei Yuan[2], and Guang-Can Guo[1]
[1] *Key Laboratory of Quantum Information, University of Science and Technology of China, Chinese Academy of Sciences, Hefei, Anhui 230026, People's Republic of China*
[2] *Center for Astrophysics, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China*

By investigating the evolution of a moving spin-1/2 Dirac electron coupled with an a background magnetic noise, we demonstrate that the effects of special relativity will significantly modify the decoherence properties of the spin state. The dephasing could be much suppressed, and for sufficiently long time the decoherence even seems to halt. This interesting phenomenon stems from the *dressed environment* induced by special relativity.

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I. INTRODUCTION

The integration of quantum mechanics and information theory gave birth to the theory of quantum information. As another fundamental part of modern physics theories, relativity theory also has significant interrelationship with quantum mechanics and information theory [1, 2]. One intriguing example is the relativistic thermodynamics, which was renewed when quantum properties of black holes [3] were discovered. The thermodynamics of moving bodies [4] demonstrate that probability distributions, which is relevant to Shannon entropy information, depend on the inertial frame. Most recently, the relationship of relativity theory to quantum information theory has attracted increasing interest. Since Peres, Scudo and Terno find that the spin Von Neumann entropy is not Lorentz invariant [5], the effects of Lorentz boosts on quantum states and then quantum entanglement [6, 7, 8, 9, 10, 11, 12, 13] have been widely investigated. Most of these works about relativistic quantum information theory (RQIT) concentrated on the quantum state itself. RQIT maybe necessary in future practical experiments. It has been shown that the fidelity of quantum teleportation with a uniformly accelerated partner is reduced due to Davies-Unruh radiation [12]. Other possible applications include quantum clock synchronization [14], quantum-enhanced communication [15, 16] and global positioning [17].

Quantum decoherence is closely related to several fundamental problems in quantum mechanics, e.g. quantum measurement and quantum to classical transition [18]. Therefore, the investigation of decoherence in combination with special relativity is naturally of interest and importance [19], which may help gain new insights into the fundamental issues of modern theoretic physics. It is known that quantum systems coupled with an external environment will suffer from inevitable decoherence, which is the most severe obstacle to implementing quantum computation. A famous strategy to fight against decoherence is quantum dynamical decoupling by applying different control operations, including spin echo and bang-bang control [20, 21].

Given a single spin-1/2 Dirac electron with the rest mass $m>0$, we could realize the qubit by the spin up and down along the $\hat{\mathbf{z}}$ direction. If the Dirac electron is coupled with an noisy magnetic environment, the coherence of the spin-qubit will be lost. In conventional quantum information theory (CQIT), people always assume that the central quantum system is at rest. In this paper, we present the time evolution of the spin state of a *moving* Dirac electron. It can be seen that the decoherence properties is significantly modified by special relativity. We demonstrate that the dressed environment induced by special relativity will suppress the spin decoherence. One should note that, the problem we consider here is different from the gravitational decoherence [22], which results from quantum metric fluctuations and Unruh effect.

The structure of this paper is as follows. In Sec. II we demonstrate the spin dephasing of a rest electron. In Sec. III the influences of special relativity on the spin decoherence properties is investigated by presenting its dynamical behavior in the operator-sum representation form. In Sec. IV are conclusions and some discussions.

II. SPIN DEPHASING OF A REST ELECTRON

We start by considering the decoherence process for the spin degree of freedom of an electron with a background magnetic noise. In CQIT, the electron is always assumed to be at rest. Since the spin magnetic moment of the electron is $\mu=\frac{e\hbar}{2mc}$, where e is the magnitude of the electronic charge, its interaction with a magnetic field $\mathbf{B}=\nabla\times\mathbf{A}$ is described by the following simple Hamiltonian

$$H_I=\mu\hat{\sigma}\cdot\mathbf{B} \tag{1}$$

where $\hat{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ are Pauli matrices. If the noisy background magnetic field is $\mathbf{B}=B\hat{\mathbf{z}}$ in the $\hat{\mathbf{z}}$ direction with Gaussian probability distribution $\eta(B)=$

*Electronic address: zwzhou@ustc.edu.cn

$\exp(-B^2/2\kappa^2)/\sqrt{2\pi}\vartheta$, and under quasi-static approximation [23], we can write the spin state of the electron at time t as a completely positive map with an operator-sum representation

$$\mathcal{E}(\rho) = \lambda_0(t)\rho + \lambda_1(t)\sigma_z\rho\sigma_z \tag{2}$$

where ρ is the initial spin state, and the parameters $\lambda_0(t) + \lambda_1(t) = 1$, $\lambda_0(t) - \lambda_1(t) = e^{-\gamma t^2} = \int_{-\infty}^{\infty} e^{-i2\mu Bt}\eta(B)dB$ with $\gamma = 2\vartheta^2\mu^2$. Here we have set $\hbar = 1$ for simplicity. This kind of decoherence model, named dephasing, is very important and has been widely investigated in CQIT [24]. In the dephasing process, the diagonal elements of the spin density matrix remain unchanged. The decoherence is reflected by the off-diagonal elements, which will decay exponentially as $\rho_{\uparrow\downarrow}(t) = \rho_{\uparrow\downarrow}e^{-\gamma t^2}$ until $\rho_{\uparrow\downarrow}(t) \to 0$ in the long time limit $\gamma t^2 \gg 1$, i.e. the spin state becomes classically mixed.

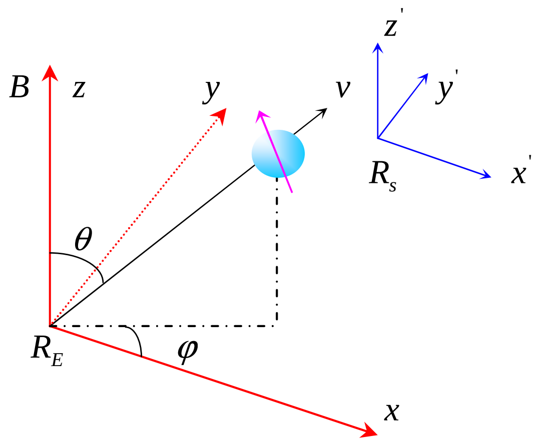

FIG. 1: (Color online) A spin-1/2 Dirac electron, at rest in the moving inertial frame R_S with the velocity $\mathbf{v}$, is coupled with the background magnetic noise in the $\hat{\mathbf{z}}$ direction of the rest frame R_E.

III. SPIN DECOHERENCE OF A MOVING ELECTRON

To deal with the relativistic Dirac electron moving at a constant velocity $\mathbf{v} = (v\sin\theta\cos\varphi, v\sin\theta\sin\varphi, v\cos\theta)$ relative to the rest frame R_E, we should adopt the Dirac equation for the electron in external homogeneous static fields. After choosing a suitable reference frame R, the Dirac Hamiltonian in Foldy-Wouthuysen representation [25] is

$$\begin{aligned} H_D &= H_p + H_{SB} \\ H_p &= mc^2 + \frac{1}{2m}(\hat{\mathbf{p}} + \frac{e}{c}\mathbf{A})^2 - \frac{\hat{\mathbf{p}}^4}{8m^3c^2} \\ H_{SB} &= \mu\hat{\sigma}\cdot\mathbf{B} - \frac{\mu}{2mc}\hat{\sigma}\cdot(\mathbf{E}\times\hat{\mathbf{p}}) \end{aligned} \tag{3}$$

where $\hat{\mathbf{p}} = -i\hbar\nabla$, and $\mathbf{E} = -\nabla\phi$ represents the electric field. The above Hamiltonian is a non-relativistic expansion to order v_p^2/c^2, where v_p is the relative velocity of the electron in the reference frame R. It is convenient for us to investigate the dynamical properties of the Dirac electron in the the moving inertial frame R_S with the velocity $\mathbf{v}$ relative to the rest frame R_E, in which the electron is at rest, i.e. $v_p = 0$. By virtue of the Foldy-Wouthuysen representation, we only need to consider the positive energy states. For the spin and momentum eigen state $|\mathbf{p}, s\rangle$, of which $|\mathbf{p}\rangle \sim e^{i\mathbf{p}\cdot\mathbf{r}}$, $H_p e^{i\mathbf{p}\cdot\mathbf{r}} = \varepsilon_p e^{i\mathbf{p}\cdot\mathbf{r}}$. After time t, the evolution of the state is $|\mathbf{p}, s\rangle \to e^{i\phi_p(t)}|\mathbf{p}\rangle e^{-iH_{SB}t}|\mathbf{s}\rangle$. The momentum phase $\phi_p(t)$ becomes trivial when tracing out the momentum degree of freedom.

In the rest frame R_E, the magnetic field is $\mathbf{B} = B\hat{\mathbf{z}}$ in the $\hat{\mathbf{z}}$ direction, thus the fields viewed in the moving frame R_S can be obtained according to the Lorentz transformations [26] as follows

$$\begin{aligned} E'_\perp &= \cosh\xi(E_\perp + \frac{\mathbf{v}}{c}\times\mathbf{B})_\perp, \quad E'_{\shortparallel} = E_{\shortparallel} \\ B'_\perp &= \cosh\xi(B_\perp - \frac{\mathbf{v}}{c}\times\mathbf{E})_\perp, \quad B'_{\shortparallel} = B_{\shortparallel} \end{aligned} \tag{4}$$

where $\shortparallel$ and $\perp$ mean parallel and perpendicular to $\mathbf{v}$, the rapidity ξ is defined as $\cosh\xi = 1/(1 - v^2/c^2)^{1/2}$. After some straightforward calculations, and note that $\mathbf{p} = \mathbf{0}$, we get the effective Hamiltonian for the spin degree of freedom of the Dirac electron $H_{SB}^{'} = \mu\hat{\sigma}\cdot\mathbf{B}'$, where $B'_x = B(1 - \cosh\xi)\cos\theta\sin\theta\cos\varphi$, $B_y^{'} = B(1 - \cosh\xi)\cos\theta\sin\theta\sin\varphi$, and $B_z^{'} = B(\cos^2\theta + \cosh\xi\sin^2\theta)$. The above effective interaction Hamiltonian means that from the viewpoint of the moving Dirac electron, the environment is different compared to the situation when the electron is at rest due to the relativity of the magnetic field. In the following, we will investigate in detail how this kind of effects will modify the dynamical properties of the spin decoherence.

The interaction with the external magnetic field $\mathbf{B}'$ will introduce a rotation on the spin by δ about the $\hat{\mathbf{n}} = (n_x, n_y, n_z)$ axis as

$$U(B,t) = \exp(-i\frac{\delta}{2}\hat{\sigma}\cdot\hat{\mathbf{n}}) \tag{5}$$

where $n_i = B'_i/B^{'}$, $i = x, y, z$, with $B^{'} = (B_x'^2 + B_y'^2 + B_z'^2)^{1/2} = \kappa B$, $\kappa = (\cos^2\theta + \cosh^2\xi\sin^2\theta)^{1/2}$, and the rotation angle is $\delta = 2\mu t B^{'} = 2\kappa\mu t B$. In the similar way, the noisy background magnetic field B is with Gaussian probability distribution, and under quasi-static approximation [23], we can write the spin state of the moving Dirac electron at time t as

$$\rho(t) = \int_{-\infty}^{\infty}\rho(B,t)\eta(B)dB \tag{6}$$

where $\rho(B,t) = U(B,t)\rho U^\dagger(B,t)$ and ρ denotes the initial spin state.

We first examine the diagonal elements by calculating $\rho_{\uparrow\uparrow}(t)$. According to Eq.(5), it is easy for us to write $\rho_{\uparrow\uparrow}(B,t) = \rho_{\uparrow\uparrow} - 2\Delta\rho_{\uparrow\uparrow}\sin^2\frac{\delta}{2} + \frac{i}{2}(\rho_{\uparrow\downarrow}e^{i\varphi} - \rho_{\downarrow\uparrow}e^{-i\varphi})(n_x^2 +$

$n_y^2)^{1/2}\sin\delta$, where $\Delta\rho_{\uparrow\uparrow}=\frac{1}{2}[\eta(\rho_{\uparrow\uparrow}-\rho_{\downarrow\downarrow})-\chi(\rho_{\uparrow\downarrow}e^{i\varphi}+\rho_{\downarrow\uparrow}e^{-i\varphi})]$ with two modulation factors defined as $\eta=(1-n_z^2)$ and $\chi=n_z(n_x^2+n_y^2)^{1/2}$. After integrating over the magnetic field B with Gaussian probability distribution, we obtain

$$\rho_{\uparrow\uparrow}(t)=\rho_{\uparrow\uparrow}-\Delta\rho_{\uparrow\uparrow}(1-e^{-\gamma' t^2}) \tag{7}$$

where $\gamma'=\kappa^2\gamma=2\kappa^2\vartheta^2\mu^2$. The first term is just the same as the dephasing process in CQIT, i.e. $v=0$. However, the second item in Eq.(7) indicates that the diagonal elements will change as time, which is a different decoherence source introduced by the effects of special relativity. In the long time limit $\gamma' t^2\gg 1$, the spin up population will decrease $\Delta\rho_{\uparrow\uparrow}$.

Now we turn to the off-diagonal elements, in the similar way we can obtain $\rho_{\uparrow\downarrow}(B,t)=\rho_{\uparrow\downarrow}e^{-i\delta}+2\Delta\rho_{\uparrow\downarrow}\sin^2\frac{\delta}{2}+\frac{i}{2}[(\rho_{\uparrow\uparrow}-\rho_{\downarrow\downarrow})(n_x-in_y)+2\rho_{\uparrow\downarrow}(1-n_z)]\sin\delta$, where $\Delta\rho_{\uparrow\downarrow}=\frac{1}{2}[\chi(\rho_{\uparrow\uparrow}-\rho_{\downarrow\downarrow})e^{-i\varphi}+\eta(\rho_{\uparrow\downarrow}+\rho_{\downarrow\uparrow}e^{-i2\varphi})]$. After integrating over the Gaussian magnetic field B, we obtain

$$\rho_{\uparrow\downarrow}(t)=\rho_{\uparrow\downarrow}e^{-\gamma' t^2}+\Delta\rho_{\uparrow\downarrow}(1-e^{-\gamma' t^2}) \tag{8}$$

The first term of $\rho_{\uparrow\downarrow}(t)$ is in a similar form of exponential decay, except that the decay rate changes to $\gamma'=\kappa^2\gamma\geq\gamma$. The effects of special relativity is also reflected by the second term, which implies that when $\gamma' t^2\gg 1$, $\rho_{\uparrow\downarrow}(t)\to\Delta\rho_{\uparrow\downarrow}$ that is a saturation value other than zero, i.e. the off-diagonals will not vanish.

From the evolution of diagonal and off-diagonal elements in Eqs.(7,8), we can establish a physical picture of the above analyses by expressing the spin density matrix at time t in the operator-sum representation

$$\mathcal{E}_m(\rho)=p_0\rho+p_1\sigma_z\rho\sigma_z-\varepsilon\sigma_z\rho\sigma_z+\sum_{i=1}^{2}F_i\rho F_i^\dagger \tag{9}$$

where $p_0=(1+e^{-\gamma' t^2})/2$, $p_1=(1-e^{-\gamma' t^2})/2$ and $\varepsilon=p_1(\eta+\chi)$. The operators $\{F_i\}$ are $F_1=[p_1(\eta-\chi)]^{1/2}(\cos\varphi\sigma_x+\sin\varphi\sigma_y)$ and $F_2=(p_1\chi)^{1/2}(\sigma_z+\cos\varphi\sigma_x+\sin\varphi\sigma_y)$. If the velocity of the electron $v=0$, the modulation factor $\eta=\chi=0$, and the above results reduce to the pure dephasing of a rest spin. When we consider a moving electron, the first two terms in Eq.(9) is similar to pure dephasing. However, the decay rate of the off-diagonal elements is amplified by the factor $\kappa^2\geq 1$. The suppression of dephasing stems from the third term $-\varepsilon\sigma_z\rho\sigma_z$. The other two operators F_1 and F_2 in Eq.(9) represent different decoherence mechanisms which makes the spin suffer from other decoherence than pure dephasing. We could also formulate the evolution of the spin state of the moving electron as in the dressed environment

$$\rho(t)=V^\dagger\left\{\int_{-\infty}^{\infty}[\exp(-i\frac{\delta_0}{2}\sigma_z)]^\kappa(V\rho V^\dagger)[\exp(i\frac{\delta_0}{2}\sigma_z)]^\kappa\eta(B)dB\right\}V \tag{10}$$

where $V=\exp[-i\frac{\phi}{2}\hat{\sigma}\cdot(\hat{\mathbf{n}}\times\hat{\mathbf{z}})]$ is the dressing transformation, ϕ is the angle between the axes $\hat{\mathbf{n}}$ and $\hat{\mathbf{z}}$. From the viewpoint of qubit, after the control operation V, the initial state ρ will undergo pure dephasing in the dressed Hilbert space, the final reverse operation $V^\dagger$ will recover some coherence information in the original Hilbert space.

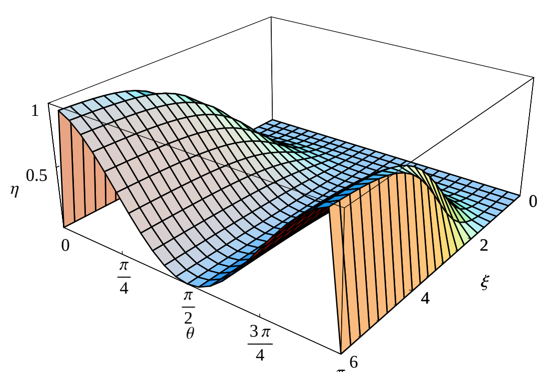

FIG. 2: (Color online) Decoherence modulation factor as a function of the moving rapidity and angle: η *vs.* ξ and θ.

The modulation factors η and χ are the key ingredients that characterize the influence of special relativity. Fig 2 shows the modulation factor η as a function of the rapidity ξ and θ. For a given rapidity ξ, the modulation factor $\eta=(\cosh\xi-1)^2(1-\cos^2 2\theta)/2[(\cosh^2\xi+1)-(\cosh^2\xi-1)\cos 2\theta]$. We are interested in the maximum value of the modulation factor η. Thus we consider the first order partial differential equation $\partial\eta/\partial(\cos 2\theta)=0$, which leads to $\cos 2\theta=(\cosh\xi-1)/(\cosh\xi+1)$, and the corresponding maximum value of η is

$$\eta_{\max}=(\frac{\cosh\xi-1}{\cosh\xi+1})^2 \tag{11}$$

We plot the maximum value $\eta_{\max}$ for various rapidity ξ in the following Fig 3(a). It can be seen that $\eta_{\max}$ always increases monotonically as the rapidity ξ grows. In the limit $\xi\to\infty$, i.e. the velocity is close to the light velocity $v\to c$, the maximum modulation factor $\eta_{\max}$ reaches the constant value 1. For the angle θ of the velocity v that maximize the value of η, the corresponding value of the other modulation factor $\chi=n_z(n_x^2+n_y^2)^{1/2}$ is $\chi_m=2\cosh^{1/2}\xi(\cosh\xi-1)/(\cosh\xi+1)^2$. When we

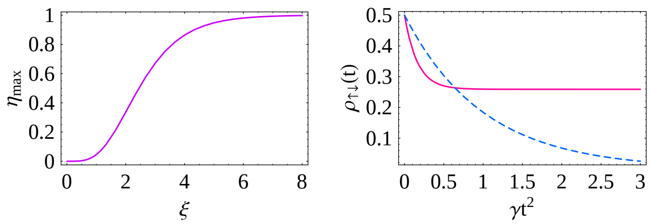

FIG. 3: (Color online) *a*. Maximum value of decoherence modulation factor as a function of rapidity: $\eta_{\max}$ *vs.* ξ; *b*. Off-diagonal element as a function of time: $\rho_{\uparrow\downarrow}(t)$ *vs.* γt^2, and the rapidity $\xi = 2.5$ (Solid), $\xi = 0$ (Dashed). The initial spin state is $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$.

consider the relativistic Dirac electron, i.e. the velocity v is comparable to the light velocity c, $\chi_m \ll \eta_{\max}$, which can be neglected in some sense.

Example To explicitly demonstrate the influence of special relativity on the spin decoherence of a moving Dirac electron, we consider a spin qubit in a fully coherent initial state $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$. We could have chosen a more general initial spin state with different relative amplitudes. However, the above state gives rise to all interesting physical features in the situation considered here. We set the velocity angle $\varphi = 0$ to maximize the modulus of the saturation value of off-diagonal elements, thus

$$\rho_{\uparrow\uparrow}(t) = \frac{1}{2}[1 + \chi(1 - e^{-\gamma' t^2})] \tag{12}$$

$$\rho_{\uparrow\downarrow}(t) = \frac{1}{2}[(1-\eta)e^{-\gamma' t^2} + \eta] \tag{13}$$

We plot the decay of the off-diagonal elements of the spin density matrix in Fig 3(b), and compare two cases when the rapidity $\xi = 2.5$ and $\xi = 0$. The velocity angle θ is set to maximize the modulation factor η. In the short time region, the off-diagonal elements decay much faster, which is due to the amplification of $\gamma \to \gamma'$ by the factor $\kappa^2 = 6.13229 > 1$. However, as the time is longer, the decay of the off-diagonal elements will be much suppressed. In particular, if the time is sufficiently long, for $\xi = 0$, the spin degree of freedom becomes completely classical; while for $\xi = 2.5$, the off-diagonal elements will reach a nonzero saturation value $\eta_{\max}/2 = 0.2589$, and the spin state becomes steady, which suggests that the decoherence process seems halting.

The above discussions are extensible to the situation of several spin-1/2 Dirac electrons coupled with one common Gaussian background magnetic noise, i.e. $H_{SB} = \mu \sum_i \hat{\sigma}^i \cdot \mathbf{B}$ [27, 28]. For example, we consider two electrons in the spin entangled state $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$ moving at the same velocity, if the electrons are at rest, the two-qubit entanglement quantified by the concurrence will decay exponentially as $\mathcal{C}(t) = \exp(-4\gamma t^2)$ [29, 30]. In order to highlight the effects of special relativity, we assume that $v \cong c$, thus the modulation factors are $\eta \cong 1$ and $\chi \cong 0$. After simple calculations, it is easy to obtain the evolution of the two-qubit entanglement as $\mathcal{C}'(t) = \exp(-4\gamma' t^2)$. Therefore, unlike the situation of a single electron, the effects of special relativity will makes $\mathcal{C}'(t) \to 0$ much more quickly.

IV. CONCLUSIONS

The existing research in the field of RQIT focused on the effects of special relativity on the *static* properties of quantum states, e.g. Von Neumann entropy and quantum entanglement. In this work, we investigate the *dynamic* properties, i.e. decoherence process of a moving Dirac electron. The decoherence mechanisms will be significantly modified due to the *dressed environment*, which leads to the intriguing phenomenon that the coherent information will be preserved even as the electron is in the noisy environment for sufficiently long time. The extension of this work to general decoherence models will enlarge the research scope of relativistic quantum information theory, and may establish fundamental connections between special relativity and quantum mechanics.

V. ACKNOWLEDGMENTS

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